Modelado de series de tiempo en entornos de big data: previsión de la (co)volatilidad con datos de alta dimensión

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Time Series Big Data

- High-Dimensional data
- High-Frequency data
- Alternative data (sentiments, texts, external covariates)

Forecasting conditional covariance matrices in high-dimensional time series via general dynamic factor models: portfolio allocation and risk measures

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Forecasting Conditional Covariance Matrices in High-Dimensional Time Series: A General Dynamic Factor Approach

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Forecasting value-at-risk and expected shortfall in large portfolios: A general dynamic factor model approach

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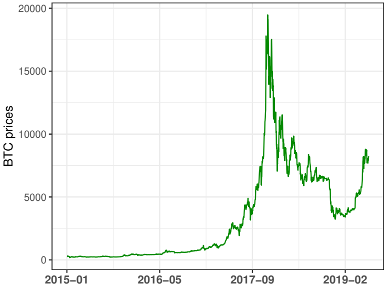
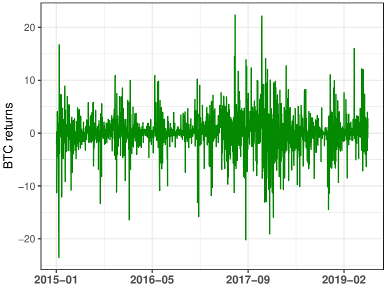
Introduction

Let P_t be the closing price at day t, the return at time t is defined by

$$r_t = (P_t - P_{t-1})/P_{t-1}.$$

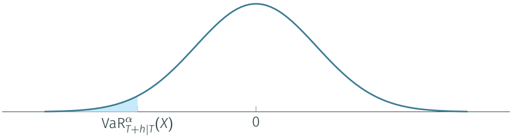




Volatility: $\sqrt{\mathbb{V}(Y_t|\mathcal{F}_{t-1})}$

VaR

Maximum possible loss for a given portfolio within a confidence interval $(1-\alpha)\times 100\%$ over a specific time horizon h



Let X_T be a given portfolio returns up to time T, the VaR h-steps-ahead at level α, denoted by $\text{VaR}^{\alpha}_{T+h|T}$, is a number such that

$$\mathbb{P}\Big(X_{T+h} < \text{VaR}^{\alpha}_{T+h|T}(X)\Big) = \alpha$$

Expected Shortfall (ES)

VaR is the maximum loss given a confidence level during a certain period, while the ES is the average loss once this loss overcomes VaR

The ES at level α over a specific time horizon h is given by

$$\text{ES}^{\alpha}_{T+h|T}(X) = \mathbb{E}\left[X_{T+h} \mid X_{T+h} \leq \text{VaR}^{\alpha}_{T+h|T}\right]$$

- In the multivariate framework, most VaR and ES estimation procedures are based on the estimation of the conditional covariance matrix.
- The (conditional) covariance matrix is an input parameter for several portfolio allocation strategies.
- Classical multivariate volatility models are helpful tools for predicting conditional covariance matrix in small and moderate dimensions, however, they badly suffer from the so-called “curse of dimensionality”.

Some alternatives to estimate the conditional covariance matrix in high dimensional data are:

- Composite likelihood
- Shrinkage
- Non-parametric approaches
- Dimension reduction techniques

Dimension reduction techniques to forecast the conditional covariance matrix:

- Principal components analysis (PCA),
- Independent component analysis (ICA),
- Conditionally uncorrelated components (CUC),
- Dynamic orthogonal components (DOC),
- Principal volatility components (PVC), etc.

However, most dimension reduction techniques are based on a static approach which is not optimal in a time series context (Hallin et al., 2018).

The general dynamic factor model

Let $\mathbf{X}_t = (X_{1t}\ X_{2t} \ldots\ X_{nt})'$, $t = 1, ...$ be a double-indexed stochastic stationary process with zero mean and finite second order moments. The GDFM is based on the decomposition

$$X_{it} = \chi_{it} + \xi_{it} \tag{1}$$

$$\chi_{it} = b_{i1}(L)u_{1t} + \ldots + b_{iq}(L)u_{qt}, \quad i \in \mathbb{N}, \quad t \in \mathbb{Z}, \tag{2}$$

where L stands for a lag operator and the unobservable $u_{i,t}$, $\chi_{i,t}$ and $\xi_{i,t}$, stand for the common shocks, common components and idiosyncratic components respectively.

Under the assumption that the space spanned by the common components is finite-dimensional, the decomposition (1) takes the static form

$$X_{it} = \underbrace{\lambda_{i1}F_{1t} + \ldots + \lambda_{ir}F_{rt}}_{\chi_{it}} + \xi_{it}, \quad r \geq q \tag{3}$$

However, this assumption rules out simple factor-loading patterns (Forni and Lippi, 2011; Forni et al., 2015, 2017) such as

$$X_{i,t} = \overbrace{\underbrace{a_i(1-\alpha_i L)^{-1}u_t}_{a_i(u_t+\alpha_i u_{t-1}+\alpha_i^2 u_{t-2}+\alpha_i^3 u_{t-3}+\ldots)}}^{\chi_{it}} + \xi_{it}. \tag{4}$$

- Forni et al. (2000) proposed a procedure that does not assume that the space spanned by the common components is finite-dimensional. However, is based on a **two-sided filter**, which is not satisfactory for forecasting.

General dynamic factor model

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- Forni et al. (2015, 2017) proposed a procedure which allows **one-sided** filter estimation and **infinite-dimensional** factor space.

Conditional covariance matrix

Forni et al. (2015, 2017) show that, $\chi_t = (\chi_{1t}\ \chi_{2t} \ldots \chi_{nt})'$ admits a block-structure autoregressive representation

$$\mathbf{A}(L)\chi_t = \mathbf{R}u_t. \tag{5}$$

where

$$\mathbf{A}(L) = \begin{bmatrix} \mathbf{A}^1(L) & 0 & \ldots & 0 \\ 0 & \mathbf{A}^2(L) & \ldots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & \ldots & \mathbf{A}^m(L) \end{bmatrix} \text{ and } \mathbf{R} = \begin{bmatrix} \mathbf{R}^1 \\ \mathbf{R}^2 \\ \vdots \\ \mathbf{R}^m \end{bmatrix}$$

with $A^k(L)$ being a $(q+1) \times (q+1)$ polynomial matrix with finite degree and R^k a $(q+1) \times q$ matrix, $n = m(q+1)$.

Proposition

Under assumptions in Barigozzi and Hallin (2020a) and additionally assuming that u_t and ξ_t are conditionally uncorrelated for any lead and lag, the conditional variance-covariance matrix of X_t is given by

$$V(X_t|\mathcal{F}_{t-1}) = \mathbf{R}\, V(\mathbf{u}_t|\mathcal{F}_{t-1})\mathbf{R}' + V(\xi_t|\mathcal{F}_{t-1}). \qquad (6)$$

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- $u_t \sim$ MGARCH.
- The conditional covariance matrix of the idiosyncratic factors can be modelled as a full or diagonal matrix, where each conditional variance is modelled independently by a GARCH-type model.

Estimation

- **Step 1.** Determine the number q of common shocks via an information criterion, for instance, using Hallin and Liška (2007).

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- **Step 2.** Randomly reorder the n observed series.
- **Step 3.** Estimate the spectral density matrix of X by

$$\widehat{\boldsymbol{\Sigma}}^X_{nT}(\theta) = \frac{1}{2\pi} \sum_{k=-M_T}^{M_T} e^{-ik\theta} \underbrace{K\left(\frac{k}{B_T}\right)}_{1-\frac{|k|}{[|\sqrt{T}|]+1}} \widehat{\boldsymbol{\Gamma}}^X_k \quad \theta \in [0, 2\pi] \qquad (7)$$

where $K(\cdot)$ is a kernel function, M_T a truncation parameter, B_T a bandwidth, and $\widehat{\boldsymbol{\Gamma}}^X_k$ the sample lag-k cross-covariance matrix.

- **Step 4.** Estimate the spectral density matrix of the common components by

$$\widehat{\mathbf{\Sigma}}^{\chi}_{nT}(\theta) := \widehat{\mathsf{P}}^{\chi}_{nT}(\theta)\widehat{\mathbf{\Lambda}}^{\chi}_{nT}(\theta)\widehat{\mathsf{P}}^{\chi*}_{nT}(\theta) \quad \theta \in [0, 2\pi]$$

where $\widehat{\mathbf{\Lambda}}^{\chi}_{nT}(\theta)$ is a $q \times q$ diagonal matrix with diagonal elements the q largest eigenvalues of $\widehat{\mathbf{\Sigma}}^{\chi}_{nT}(\theta)$ and $\widehat{\mathsf{P}}^{\chi}_{nT}(\theta)$ (with complex conjugate $\widehat{\mathsf{P}}^{\chi*}_{nT}$) is the $n \times q$ matrix with the associated eigenvectors.

- **Step 5.** By inverse Fourier transform of $\widehat{\mathbf{\Sigma}}^{\chi}_{n^*T}(\theta)$, estimate the autocovariance matrices $\widehat{\mathbf{\Gamma}}^{\chi}_{k}$ of the m sub-vectors

$$\boldsymbol{\chi}^{k}_{t} = (\chi_{(k-1)(q+1)+1,t} \cdots \chi_{k(q+1),t})', \quad k = 1, ..., m$$

of dimension $(q+1)$. Based on these, compute, after order identification, the Yule-Walker estimators $\hat{\mathsf{A}}^{k}(L)$ of the m VAR filters $\mathsf{A}^{k}(L)$ and stack them into a block-diagonal matrix $\hat{\mathsf{A}}_{n^*}(L)$. Compute $\hat{\mathsf{A}}_{n^*}(L)\mathsf{X}_{n^*t} = \widehat{\mathsf{Y}}_{n^*t} = \widehat{\mathsf{R}}_{n^*}\widehat{\mathsf{u}}_{t} + \widehat{\varepsilon}_{t}$

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- **Step 6.** Based on the first q standard principal components of $\widehat{\mathbf{Y}}_{n^*t}$, obtain estimates $\widehat{\mathbf{R}_{n^*}\mathbf{u}_t}$ of $\mathbf{R}_{n^*}\mathbf{u}_t$ and, via a Cholesky identification constraint the estimates $\widehat{\mathbf{R}}_{n^*}$ and $\widehat{\mathbf{u}}_t$ of $\mathbf{R}_{n^*}$ and $\mathbf{u}_t$; then, an estimate of the impulse-response function is $\widehat{\mathbf{B}}_{n^*}(L) := [\hat{\mathbf{A}}_{n^*}(L)]^{-1}\widehat{\mathbf{R}}_{n^*}$.

- **Step 7.** Repeat Steps 2 through 7 M times: the final estimates (denoted as $\widehat{\mathsf{R}}_n$, $\widehat{\mathsf{u}}_t$, and $\widehat{\mathsf{B}}_n$) are obtained by averaging the estimates $\widehat{\mathsf{R}}_{n^*}$, $\widehat{\mathsf{u}}_t$, and $\widehat{\mathsf{B}}_{n^*}$ associated with each iteration. Let $\widehat{\chi}_{nt} := \widehat{\mathsf{B}}_n(L)\widehat{\mathsf{u}}_t$ and $\hat{\xi}_{nt} := \mathsf{X}_{nt} - \widehat{\chi}_{nt}$.

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- **Step 8.** The estimator of $\mathrm{V}(\mathsf{X}_t|\mathcal{F}_{t-1})$ is given by

$$\widehat{\mathrm{V}}(\mathsf{X}_t|\mathcal{F}_{t-1}) := \widehat{\mathsf{R}}\widehat{\mathrm{V}}(\hat{\mathsf{u}}_t|\mathcal{F}_{t-1})\widehat{\mathsf{R}}' + \widehat{\mathrm{V}}(\hat{\xi}_t|\mathcal{F}_{t-1}). \quad (8)$$

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The consistency of the method is established under some assumptions (stationarity, the existence of a spectral density matrix, etc.). See, Forni et al. (2017), Barigozzi and Hallin (2020b), Trucíos et al. (2023).

Monte Carlo experiments

General settings:

- 500 Monte Carlo replications.
- Panel of dimension 60×1000 and 600×700.
- Three DGPs.
  - **DGP1:** Static factor model with $\mathbf{u}_t$ following a bivariate full BEKK and ξ_{it} following univariate GARCH.

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  - **DGP2:** Dynamic factor model with finite-dimensional factor space, $\mathbf{u}_t$ follows a bivariate full BEKK and ξ_{it} following univariate GARCH.
  - **DGP3:** Dynamic factor model with infinite-dimensional factor space, $\mathbf{u}_t$ follows a bivariate full BEKK and ξ_{it} following univariate GARCH.

Loss function

$$L(\hat{\Sigma}_{T+1|T}, \Sigma_{T+1|T}) = \sum_{i=1}^{N}\sum_{j=i}^{N} w(i,j)(\hat{\sigma}_{i,j} - \sigma_{i,j})^2, \quad (9)$$

where $\hat{\sigma}_{i,j}$ and $\sigma_{i,j}$ are the (i,j) elements of $\hat{\Sigma}_{T+1|T}$ and $\Sigma_{T+1|T}$. $w(i,j)$ are weights.

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where $\hat{\sigma}_{i,j}$ and $\sigma_{i,j}$ are the (i,j) elements of $\hat{\Sigma}_{T+1|T}$ and $\Sigma_{T+1|T}$. $w(i,j)$ are weights.

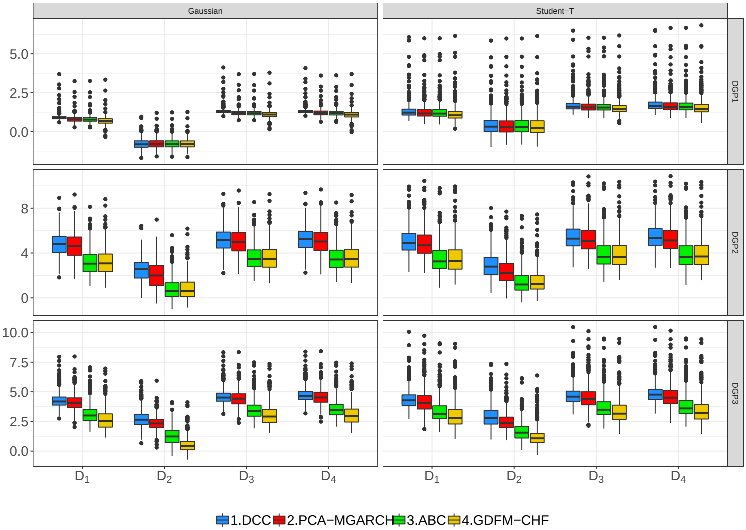
Table 1: Weights $w(i,j)$ in Equation (9), $i = 1, \ldots, N$, $j = i, \ldots, N$

D_1	$w(i,j) = 1$ $\forall i$ and j
D_2	$w(i,j) = 1$ when $i = j$ and zero otherwise
D_3	$w(i,j) = 2$ when $\hat{\sigma}_{i,j}, > h_{i,j}$; 1 otherwise
D_4	$w(i,j) = 2$ when $\hat{\sigma}_{i,j}, < h_{i,j}$; 1 otherwise

Procedures

- PCA-DCC: The static factors are extracted by using PCA and a DCC model is applied on the extracted factors. The idiosyncratic components are modelled by univariate GARCH models.
- DCC: as in Pakel et al. (2020);
- ABC-DCC: dynamic factor extraction under a finite-dimensional factor space assumption as in Alessi et al. (2009);
- GDFM-CHF: Our proposal.

Monte Carlo experiments: N = 60, T = 1000



Empirical Application I

Let H_{T+1} be the conditional covariance matrix of vector returns at time $T+1$ and $\omega = (\omega_1, \cdots, \omega_N)$ be the portfolio weights.

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We are interesting in portfolios with minimum risk subject to some constraints, *i.e.*

$$\text{Min:} \quad \omega' H_{T+1} \omega$$

subject to $\omega_i \geq 0$ and $\sum_{i=1}^{N} \omega_i = 1$

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$$\text{Min:} \quad \omega' H_{T+1} \omega$$

subject to $\omega_i \geq 0$ and $\sum_{i=1}^{N} \omega_i = 1$

This portfolio is called **the minimum variance portfolio with no short sales** and is of particular interest for investors.

- **Minimum variance portfolios**
- 656 stocks entering the composition of the S&P 500, NASDAQ100 and AMEX (only series with no missing values were considered).
- Stocks traded from January 2, 2011 through June 29, 2018 (T=1884).
- **Rolling Window scheme:** A window size of 750 days is used for estimation, which represents a concentration ratio of $656/750 = 0.875$; the out-of-sample period was set to 1134 days.

At time $t = 750, \ldots, 1883$ (1134 time points), the one-step ahead conditional covariance matrix is estimated and used to obtain the optimal portfolio allocation weights.

$$\widehat{\boldsymbol{\omega}}_{t+1|t} = \underset{\boldsymbol{\omega}}{\text{argmin}}\, \boldsymbol{\omega}' \widehat{\Sigma}_{t+1|t} \boldsymbol{\omega},$$

subject to the restrictions $\omega_i \geq 0$ and $\sum_{i=1}^{n} \omega_i = 1$.

At time $t = 750, \ldots, 1883$ (1134 time points), the one-step ahead conditional covariance matrix is estimated and used to obtain the optimal portfolio allocation weights.

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subject to the restrictions $\omega_i \geq 0$ and $\sum_{i=1}^{n} \omega_i = 1$.

Then, the resulting (out-of-sample) portfolio return is given by

$$r_{p,t+1} := \sum_{i=1}^{n} \widehat{\omega}_{i;t+1|t} r_{i,t+1}$$

To evaluate the out-of-sample portfolio performance, we use three measures:

- **SD**: Annualized standard deviation (standard deviation of the out-of-sample portfolio returns $\times\sqrt{252}$);
- **IR**: Information ratio ($\text{IR} := \text{AV}/\text{SD}$), with AV being the annualized average portfolio (average out-of-sample portfolio returns $\times$ 252);
- **SR**: Sortino ratio.

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- IR: Information ratio (IR $:=$ AV/SD), with AV being the annualized average portfolio (average out-of-sample portfolio returns $\times$ 252);
- SR: Sortino ratio.

To assess the statistical differences between our proposal and its competitors we perform the test of Ledoit and Wolf (2011) (for SD) and Ledoit and Wolf (2008) (for IR).

Competitors:

- **1/n**: equal-weighted portfolio strategy advocated by ?.
- **POET**: the estimator proposed by ?.
- **NL**: the nonlinear shrinkage estimator of ?.
- **RM2006**: the RiskMetrics 2006 methodology of Zumbach (2007).
- **DCC**: the DCC model with composite likelihood of Pakel et al. (2020).
- **OGARCH**: the orthogonal GARCH model of Alexander and Chibumba (1996).
- **GPVC**: generalized principal volatility components of Li et al. (2016).

Competitors:

- PCA4TS: principal component analysis for second-order stationary vector time series of Chang et al. (2018).
- PCA-MGARCH: as used in our simulation study.
- ABC: the procedure of Alessi et al. (2009) based on the general dynamic factor model with finite-dimensional factor space.
- 2GDFM4V: the two-step GDFM procedure for volatilities proposed by Barigozzi and Hallin (2016, 2017, 2020a); does not consider conditional cross-covariances.

Our method outperform all previously mentioned procedures and performs statistically equal than **DCC-NL** and **AFM-DCC-NL**.

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- **DCC-NL**: as proposed by Engle et al. (2017); combines the DCC and NL procedures.
- **AFM-DCC-NL**: approximate factor model as in **?** but with unobservable common components obtained by classical PCA. The covariance matrix of the idiosyncratic components is estimated using the DCC-NL procedure.

Table 2: Out-of-sample portfolio performance

	SD	IR	SR
1/n	11.5067 (14)	0.5015 (14)	0.6834 (14)
POET	4.6116 (9)	1.2146 (7)	1.6741 (7)
NL	4.6152 (10)	1.0217 (11)	1.4249 (11)
RM2006	4.5446 (7)	1.2327 (6)	1.7241 (6)
DCC	5.9901 (12)	1.1502 (8)	1.6262 (8)
DCC-NL	**3.9358 (2)**	**1.8371 (2)**	2.6227 (2)
OGARCH	4.4551 (5)	1.1051 (10)	1.5616 (10)
GPVC	4.5889 (8)	1.0022 (12)	1.4077 (12)
PCA4TS	4.7256 (11)	1.1364 (9)	1.6032 (9)
PCA-MGARCH	4.4111 (4)	1.4965 (4)	2.0891 (4)
AFM-DCC-NL	**3.9472 (3)**	**1.9764 (1)**	2.8974 (1)
ABC	4.5313 (6)	1.4404 (5)	1.9677 (5)
2GDFM4V	10.2431 (13)	0.7992 (13)	1.0997 (13)
GDFM-CHF-DCC	**3.9205 (1)**	**1.8188 (3)**	2.4109 (3)

VaR and ES

Let $\mathbf{r}_t = \mathbf{H}_t^{1/2}\boldsymbol{\epsilon}_t$, be the $N-$dimensional vector of individual returns at time t

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- **Step 10.** Using the historical returns $\mathbf{r}_1, \ldots, \mathbf{r}_T$ to obtain the estimators $\widehat{\mathbf{H}}_t := \widehat{\mathbf{V}}^{\mathbf{r}_N}_{t|t-1}$ of the conditional covariance matrices along Steps 1-9, compute the filtered (or *devolatilized*) return vectors $\hat{\boldsymbol{\epsilon}}_t := \widehat{\mathbf{H}}_t^{-1/2}\mathbf{r}_t$, where $\widehat{\mathbf{H}}_t^{1/2}$ follows from the lower triangular Cholesky factorization of $\widehat{\mathbf{H}}_t$ at time $t = 1, \ldots, T+1$.

- **Step 11.** Generate a bootstrap sample $\boldsymbol{\epsilon}_1^*, \ldots, \boldsymbol{\epsilon}_B^*$ of size B from the devolatilized return vectors $\hat{\boldsymbol{\epsilon}}_t$, $T = 1, \ldots, T+1$ and construct B one-step-ahead return vectors $\mathbf{r}_{T+1}^{i*} := \widehat{\mathbf{H}}_{T+1}^{1/2}\boldsymbol{\epsilon}_i^*$; this yields B simulated one-step-ahead portfolio return forecasts $R_{T+1}^{i*} := \boldsymbol{\omega}'\mathbf{r}_{T+1}^{i*}$, for $i = 1, \ldots, B$ for portfolio weights $\boldsymbol{\omega} = (\omega_1, \ldots \omega_N)'$.

- **Step 12.** The one-step-ahead forecasts of $\mathrm{VaR}^{\alpha}_{\mathrm{T+1}}$ and $\mathrm{ES}^{\alpha}_{T+1}$ are

$$\widehat{\mathrm{VaR}}^{\alpha}_{\mathrm{T+1}} := \widehat{F}^{-1}_{R^{*}_{T+1}}(\alpha), \tag{10}$$

and

$$\widehat{\mathrm{ES}}^{\alpha}_{\mathrm{T+1}} := \sum_{i=1}^{B} \frac{R^{i*}_{T+1}\mathbb{I}\left[R^{i*}_{T+1} < \widehat{\mathrm{VaR}}^{\alpha}_{\mathrm{T+1}}\right]}{\sum_{t=1}^{B}\mathbb{I}\left[R^{i*}_{T+1} < \widehat{\mathrm{VaR}}^{\alpha}_{\mathrm{T+1}}\right]}, \tag{11}$$

respectively, where $\widehat{F}^{-1}_{R^{*}_{T+1}}(\alpha)$ is the α-quantile of the empirical distribution of the simulated one-step-ahead portfolio returns $R^{1*}_{T+1}, \ldots, R^{B*}_{T+1}$ and $\mathbb{I}[\,\cdot\,]$ denotes the indicator function.

Empirical Application II

Empirical Application

- 652 stocks used in the composition of the S&P500, NASDAQ-100 and AMEX indexes
- From January 3, 2012 to July 1, 2020 ($T = 2136$ observations)
- A rolling window approach with window size of T= 750 days (concentration ratio of $652/750 = 0.87$).
- 1386 days in the out-of-sample period 8from December 29, 2014 through July 1, 2020).
- In each window, the one-step-ahead VaR and ES is estimated

Alternative models:

- DCC composite likelihood (Pakel et al., 2020)
- RiskMetrics 2006 methodology (Zumbach, 2007)
- ABC (Alessi et al., 2009)

Classical backtesting procedures (calibration tests and scoring functions) and scoring functions are used to evaluate the VaR and ES accuracy.

Calibration tests:

Test	Proposed by	Used to evaluate
Unconditional coverage (UC)	Kupiec (1995)	VaR
Conditional coverage (CC)	Christoffersen (1998)	VaR
Dynamic quantile (DQ)	Engle and Manganelli (2004)	VaR
VaR quantile regression (VQ)	Gaglianone et al. (2011)	VaR
Exceedance residuals (ER)	McNeil and Frey (2000)	ES and VaR
Conditional calibration (CoC)	Nolde et al. (2017)	ES and VaR
Exceedance shortfall regression (ESR)	Bayer and Dimitriadis (2020)	ES

Table 3: Calibration tests used to evaluate VaR and ES accuracy.

$$H_0 : \text{The VaR/ES is correctly specified}$$

Scoring functions:

For the VaR:

$$S(\mathrm{VaR}_{\mathrm{t}}^{\alpha}, r_t) = (\alpha - \mathbb{I}[r_t \leq \mathrm{VaR}_{\mathrm{t}}^{\alpha}])(G(r_t) - G(\mathrm{VaR}_{\mathrm{t}}^{\alpha})), \tag{12}$$

For the VaR and ES (jointly):

$$\begin{aligned} S\big((\mathrm{VaR}_t^{\alpha}, \mathrm{ES}_t^{\alpha}), r_t\big) =& \big(\mathbb{I}[r_t \leq \mathrm{VaR}_t^{\alpha}] - \alpha\big) G_1(\mathrm{VaR}_t^{\alpha}) - \mathbb{I}[r_t \leq \mathrm{VaR}_t^{\alpha}]\, G_1(r_t) \\ & + G_2(\mathrm{ES}_t^{\alpha})\big(\mathrm{ES}_t^{\alpha} - \mathrm{VaR}_t^{\alpha} + \mathbb{I}[r_t \leq \mathrm{VaR}_t^{\alpha}]\big)(\mathrm{VaR}_t^{\alpha} - r_t)/\alpha \\ & + G_3(\mathrm{ES}_t^{\alpha}) + G_4(r_t), \end{aligned} \tag{13}$$

		Hits	UC	CC	DQ	VQ	ER	CoC	ESR_1	ESR_2	ESR_3
			Calibration tests								
1%	RM2006	0.60	0.161	0.051	0.091	0.331	0.493	0.219	0.344	0.218	0.706
	DCCc	1.70	0.024	0.000	0.000	0.434	0.404	0.106	0.516	0.505	0.278
	ABC	1.90	0.003	0.001	0.000	0.258	0.024	0.052	0.016	0.043	0.036
	GDFM-CHF	1.40	0.189	0.224	0.266	0.726	0.564	0.493	0.500	0.576	0.180
2.5%	RM2006	1.30	0.002	0.000	0.000	0.001	0.522	0.000	0.009	0.011	0.972
	DCCc	3.00	0.221	0.003	0.000	0.000	0.087	0.129	0.013	0.165	0.018
	ABC	3.10	0.166	0.014	0.000	0.420	0.001	0.227	0.016	0.325	0.256
	GDFM-CHF	2.70	0.689	0.602	0.362	0.437	0.230	0.549	0.421	0.834	0.135
5%	RM2006	2.50	0.000	0.000	0.000	0.000	0.416	0.000	0.000	0.438	0.993
	DCCc	5.50	0.416	0.000	0.000	0.343	0.037	0.077	0.029	0.035	0.007
	ABC	5.50	0.416	0.024	0.000	0.588	0.000	0.324	0.906	0.010	0.409
	GDFM-CHF	5.60	0.351	0.147	0.207	0.354	0.369	0.405	0.296	0.766	0.072

Table 4: One-step-ahead VaR and ES backtesting (calibration tests).

		Avg. Scoring functions			
		QL	FZG	NZ	AL
1%	RM2006	0.038	0.673	1.850	2.178
	DCCc	0.039	0.670	1.863	2.181
	ABC	0.042	0.698	1.991	2.344
	GDFM-CHF	0.032	0.653	1.730	2.060
2.5%	RM2006	0.075	0.675	1.654	1.975
	DCCc	0.075	0.666	1.642	1.951
	ABC	0.077	0.677	1.680	2.007
	GDFM-CHF	0.066	0.631	1.535	1.816
5%	RM2006	0.123	0.681	1.493	1.797
	DCCc	0.121	0.666	1.473	1.759
	ABC	0.119	0.668	1.478	1.773
	GDFM-CHF	0.110	0.630	1.396	1.649

Table 5: One-step-ahead VaR and ES backtesting (scoring functions). Shaded cells stand for the smallest scoring function.

Additive outliers

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Outliers can deteriorate the out-of-sample performance of both VaR and ES (Grané and Veiga, 2010; Boudt et al., 2013; Trucíos et al., 2017, 2018; Trucíos et al., 2018)

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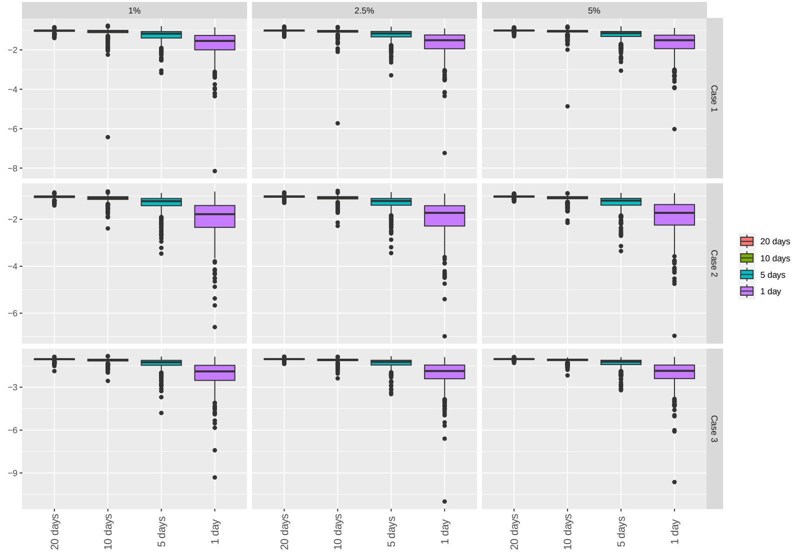
Monte Carlo Simulations

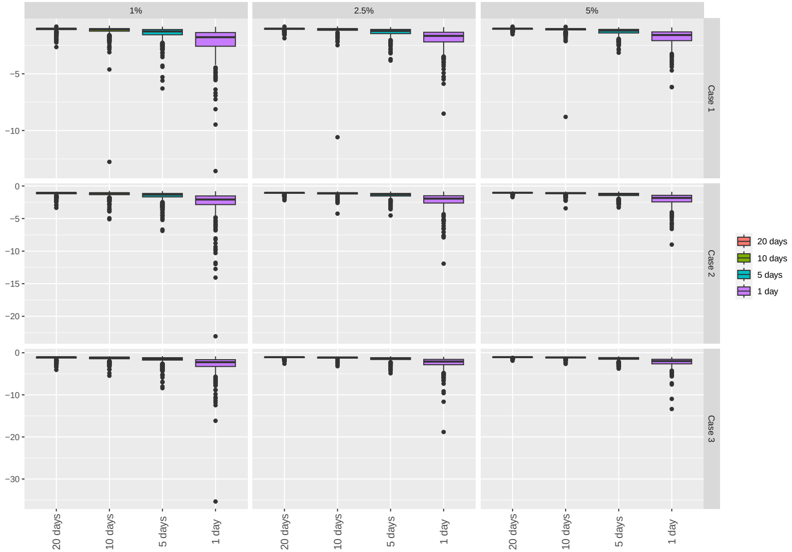
- Case 1: 10% of series contaminated by outliers.
- Case 2: 25% of series contaminated by outliers.
- Case 3: 50% of series contaminated by outliers

Series were contaminated by one isolated outlier of size equal to 5 times the uncontaminated standard deviation. Outliers are located at the end of the sample period, 5, 10 and 20 days before the end of the sample period.

Additive outliers: VaR



Additive outliers: ES



- **Step 1*.** Determine the number q of common using the robust procedure of Trucíos et al. (2021).

- **Step 1*.** Determine the number q of common using the robust procedure of Trucíos et al. (2021).
- **Step 9*.** Estimate $\mathrm{V}(\mathsf{X}_t|\mathcal{F}_{t-1})$ via

$$\widehat{\mathrm{V}}(\mathsf{X}_t|\mathcal{F}_{t-1}) := \widehat{\mathsf{R}}\widehat{\mathrm{V}}(\hat{\mathsf{u}}_t|\mathcal{F}_{t-1})\widehat{\mathsf{R}}' + \widehat{\mathrm{V}}(\hat{\xi}_t|\mathcal{F}_{t-1}). \tag{14}$$

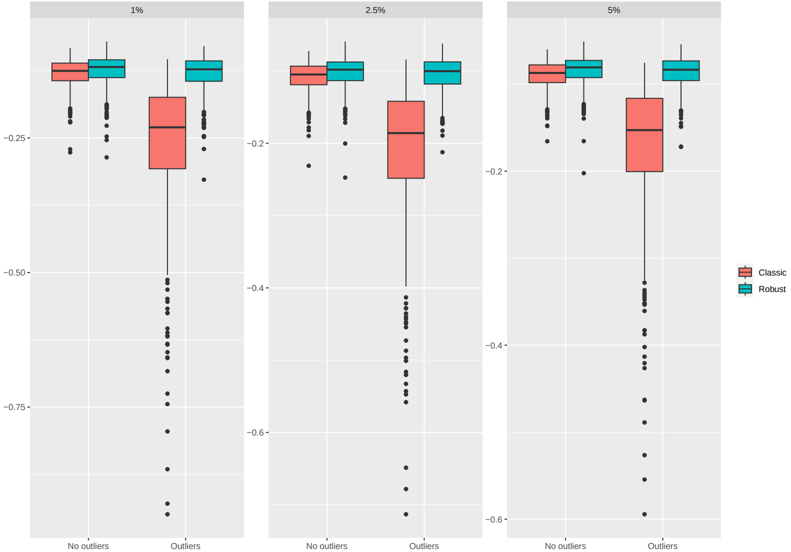
  by using the robust procedures of Trucíos et al. (2018) and Trucíos et al. (2017), for $\widehat{\mathrm{V}}(\hat{\mathsf{u}}_t|\mathcal{F}_{t-1})$ and $\widehat{\mathrm{V}}(\hat{\xi}_t|\mathcal{F}_{t-1})$, respectively.

- **Step 11*.** Generate a bootstrap sample $\boldsymbol{\epsilon}_1^*, \ldots, \boldsymbol{\epsilon}_B^*$ of size B from the devolatilized return vectors $\hat{\boldsymbol{\epsilon}}_t$, $T = 1, \ldots, T+1$ and construct B one-step-ahead return vectors $\mathbf{r}_{T+1}^{i*} := \widehat{\mathbf{H}}_{T+1}^{1/2} \rho(\epsilon_i^*)$, with

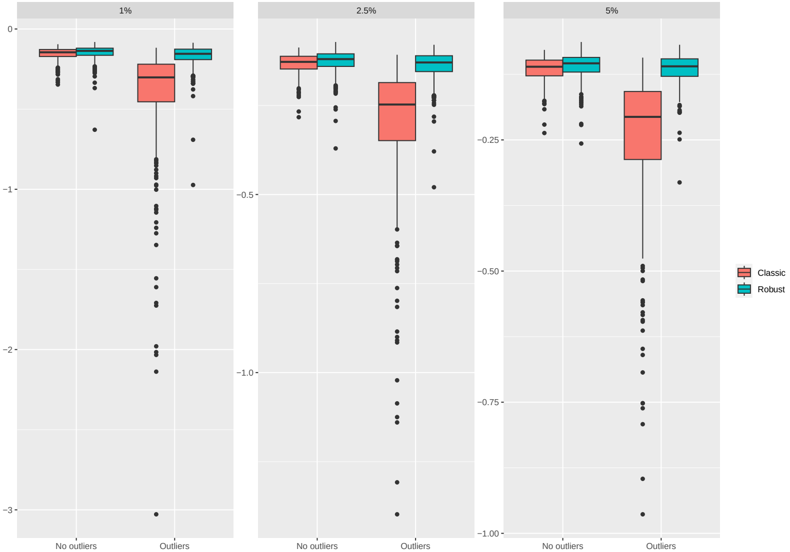
$$\rho(\mathbf{x}_i) = \begin{cases} \mathbf{x}_i & \text{if } d_i^2 < c, \\ \mathbf{x}_i^* & \text{if } d_i^2 \geq c \end{cases}$$

where $\mathbf{x}_i^*$ being another bootstrap sample from F

Additive outliers: VaR



ADDITIVE OUTLIERS: ES



Conclusions

Conclusions

- A new procedure to forecast the conditional covariance matrix in high-dimensional data with good finite sample properties and consistency results, is proposed.
- A filtered historical simulation method with the conditional covariance estimator of Trucíos et al. (2023) to estimate the VaR and ES is suggested.
- A robust-to-outliers alternative is also proposed. Experiments with simulated data are encouraging.

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