A new proposal to forecast high-dimensional conditional covariance matrices via General Dynamic Factor Models

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Introduction

Let P_t be the closing price of a given asset at time t. Returns are defined as

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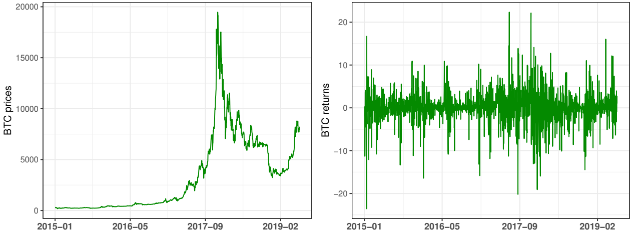

Figure 1: Prices (left panel) and returns (right panel) of Bitcoin

- Volatility (i.e, conditional standard deviations) is very important in finance.
- In a univariate context, there several option to forecast the volatility: GARCH-type, GAS, Stochastic volatility.
- In a multivariate context (for small or moderate dimensions), there are some approaches mainly based on multivariate GARCH models (MGARCH)
- In a high-dimensional context (hundred or even thousands of time series) the problems is even harder.

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Our work is placed in a high-dimensional context!

- The conditional covariance matrix plays an important role in economics and finance.

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- Classical multivariate volatility models are helpful tools for predicting conditional covariance matrix in small and moderate dimensions, but they badly suffer from the so-called “curse of dimensionality”.
- To overcome this problem, alternative procedures have been proposed in the literature.
- Some of those procedures are based on dimension reduction techniques.

Dimension reduction techniques to forecast the conditional covariance matrix:

- Principal components analysis (PCA),
- Independent component analysis (ICA),
- Conditionally uncorrelated components (CUC),
- Dynamic orthogonal components (DOC),
- Principal volatility components (PVC), etc.

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Most dimension reduction techniques are based on a static approach which is not optimal in a time series context (Hallin et al., 2018).

The general dynamic factor model

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- Sucessfully applied in several fields
- Adopted by central banks and other organizations worldwide.
- Proposed in the early 2000s.

Let $\mathbf{X}_t = (X_{1t}\ X_{2t} \ldots\ X_{nt})'$, $t = 1, \ldots$ be a double-indexed second order stationary stochastic process with zero mean and finite second order moments. The GDFM is based on the decomposition

$$X_t = \chi_t + \xi_t \tag{1}$$

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Let $\mathbf{X}_t = (X_{1t}\; X_{2t} \ldots\; X_{nt})'$, $t = 1, \ldots$ be a double-indexed second order stationary stochastic process with zero mean and finite second order moments. The GDFM is based on the decomposition

$$\mathbf{X}_t = \boldsymbol{\chi}_t + \boldsymbol{\xi}_t \tag{1}$$

$$X_{it} = \chi_{it} + \xi_{it} \tag{2}$$

$$\chi_{it} = b_{i1}(L)u_{1t} + \ldots + b_{iq}(L)u_{qt}, \quad i \in \mathbb{N}, \quad t \in \mathbb{Z}, \tag{3}$$

where L stands for a lag operator and the unobservable $u_{i,t}$, $\chi_{i,t}$ and $\xi_{i,t}$, stand for the common shocks, common components and idiosyncratic components respectively.

Under the assumption that the space spanned by the common components is finite-dimensional, the decomposition (2) takes the static form

$$X_{it} = \underbrace{\lambda_{i1}F_{1t} + \ldots + \lambda_{ir}F_{rt}}_{\chi_{it}} + \xi_{it}, \quad r \geq q \tag{4}$$

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However, this assumption rules out simple factor-loading patterns (Forni and Lippi, 2011; Forni et al., 2015, 2017) such as

$$X_{i,t} = \underbrace{\overbrace{a_i(1-\alpha_i L)^{-1}u_t}^{\chi_{it}}}_{a_i(u_t+\alpha_i u_{t-1}+\alpha_i^2 u_{t-2}+\alpha_i^3 u_{t-3}+\ldots)} + \xi_{it}. \tag{5}$$

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- Forni et al. (2005) proposed a procedure which allows for one-sided filter estimation. However, assume finite-dimensional factor space.
- Forni et al. (2015, 2017) proposed a procedure which allows one-sided filter estimation and infinite-dimensional factor space.

Principal Volatility Components

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Similar to PCA.

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- Hu, Y. P., & Tsay, R. S. **(2014)**. Principal volatility component analysis. Journal of Business & Economic Statistics, 32(2), 153-164.
- Li, W., Gao, J., Li, K., & Yao, Q. **(2016)**. Modeling multivariate volatilities via latent common factors. Journal of Business & Economic Statistics, 34(4), 564-573.

Principal Volatility Components (PVC)

- PCA decomposes a N-dimensional vector into N contemporaneous uncorrelated components according with the amount of variability explained by the components.
- PCA uses the sample covariance matrix.
- Hu and Tsay (2014, JBES) proposed PVC: A generalization of PCA that takes into account the dynamic dependence between the volatility processes.
- The covariance matrix is replaced by a matrix that summarizes the dynamic dependence of volatilities.
- With PVC we identify common volatility components and also components with no conditional heteroscedasticity.

Principal Volatility Components (PVC)

Let the *Cumulative Generalized Kurtosis Matrix* defined by

$$\Gamma_\infty = \sum_{\ell=1}^{\infty}\sum_{i=1}^{k}\sum_{j=1}^{m} E\left[\left(y_t'y_t - E(y_t'y_t)\right)\left(x_{ij,t-\ell} - E(x_{ij,t})\right)\right],$$

where $x_{ij,t-k}$ is a function of the cross product $y_{i,t-k}$ and $y_{j,t-k}$

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where $x_{ij,t-k}$ is a function of the cross product $y_{i,t-k}$ and $y_{j,t-k}$

Additionally,

$$\Gamma_\infty M = M\Lambda, \qquad \text{where}$$

- Λ is the diagonal matrix of ordered eigenvalues,
- $M = [m_1, ..., m_k]$ is the matrix of normalized eigenvectors and

Hu and Tsay (2014) proves that, under mild conditions, there exist r linear combination of y_t that have (ARCH effects a $k - r$ linear combination of y_t that have no ARCH effects if and only if $rank(\Gamma_\infty) = r$.

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The v-th PVC is defined as

$$z_{vt} = m_v' y_t$$

- z_{vt}, z_{ut} are (contemporaneously) uncorrelated if $\lambda_v^2 \neq \lambda_u^2$.
- z_{vt} may still be correlated with lagged values of z_{ut}.

Li et al. (2016) proposed an alternative PVC characterized only by the second moment. In this approach, the matrix Γ_∞ is replaced by

$$G = \sum_{k=1}^{g} \sum_{t=1}^{T} \omega(y_t) E^2 \left[\left(y_t y_t' - \Sigma \right) I(\|y_{t-k}\| \leq \|y_t\|) \right],$$

where $\omega(\cdot)$ is a weight function and $\| \cdot \|$ is the L_1 norm.

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The alternative procedure has the same properties of the proposal of Hu and Tsay (2014) but only requires finite second-order moments.

The matrix G is estimated in a natural way by

$$\hat{G} = \sum_{k=1}^{g} \sum_{\tau=1}^{T} \omega(y_\tau) \left[\frac{1}{T-k} \sum_{t=k+1}^{T} \left[\left(y_t y_t' - \hat{\Sigma} \right) I(\|y_{t-k}\| \leq \|y_\tau\|) \right] \right]^2 .$$

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Trucíos et al. (2019) based on Hu and Tsay (2014) and Li et al. (2016) proposes a robust-to-outliers PVC.

The new proposal

GDFM + PVC

Forni et al. (2015, 2017) show that, $\chi_t = (\chi_{1t}\ \chi_{2t} \dots \chi_{nt})'$ admits a block-structure autoregressive representation

$$\mathsf{A}(L)\chi_t = \mathsf{R}u_t. \tag{6}$$

where

$$\mathsf{A}(L) = \begin{bmatrix} \mathsf{A}^1(L) & 0 & \dots & 0 \\ 0 & \mathsf{A}^2(L) & \dots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & \dots & \mathsf{A}^m(L) \end{bmatrix} \text{ and } \mathsf{R} = \begin{bmatrix} \mathsf{R}^1 \\ \mathsf{R}^2 \\ \vdots \\ \mathsf{R}^m \end{bmatrix}$$

with $A^k(L)$ being a $(q+1) \times (q+1)$ polynomial matrix with finite degree and R^k a $(q+1) \times q$ matrix, $n = m(q+1)$ and n tends to infinity.

Proposition

Trucíos et al. (2023) under assumptions in Barigozzi and Hallin (2020) and additionally assuming that u_t and ξ_t are conditionally uncorrelated for any lead and lag, the conditional variance-covariance matrix of X_t is given by

$$V(\mathbf{X}_t|\mathcal{F}_{t-1}) = \mathbf{R}\, V(\mathbf{u}_t|\mathcal{F}_{t-1})\mathbf{R}' + V(\xi_t|\mathcal{F}_{t-1}). \quad (7)$$

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- $u_t \sim$ MGARCH.
- $\xi_t \sim$ MGARCH or univariate independent GARCH models.

Estimation procedure

- **Step 1.** Determine the number q of common shocks via an information criterion, for instance, using Hallin and Liška (2007).

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- **Step 2.** Randomly reorder the n observed series.
- **Step 3.** Estimate the spectral density matrix of $\mathbf{X}$ by

$$\widehat{\boldsymbol{\Sigma}}_{nT}^{X}(\theta) = \frac{1}{2\pi} \sum_{k=-M_T}^{M_T} e^{-ik\theta} \underbrace{K\left(\frac{k}{B_T}\right)}_{1-\frac{|k|}{[|\sqrt{T}|]+1}} \widehat{\boldsymbol{\Gamma}}_k^X \quad \theta \in [0, 2\pi] \qquad (8)$$

where $K(\cdot)$ is a kernel function, M_T a truncation parameter, B_T a bandwidth, and $\widehat{\boldsymbol{\Gamma}}_k^X$ the sample lag-k cross-covariance matrix.

- **Step 4.** Estimate the spectral density matrix of the common components by

$$\widehat{\mathbf{\Sigma}}^{\chi}_{nT}(\theta) := \widehat{\mathsf{P}}^{\chi}_{nT}(\theta)\widehat{\mathbf{\Lambda}}^{\chi}_{nT}(\theta)\widehat{\mathsf{P}}^{\chi*}_{nT}(\theta) \quad \theta \in [0, 2\pi]$$

where $\widehat{\mathbf{\Lambda}}^{\chi}_{nT}(\theta)$ is a $q \times q$ diagonal matrix with diagonal elements the q largest eigenvalues of $\widehat{\mathbf{\Sigma}}^{\chi}_{nT}(\theta)$ and $\widehat{\mathsf{P}}^{\chi}_{nT}(\theta)$ (with complex conjugate $\widehat{\mathsf{P}}^{\chi*}_{nT}$) is the $n \times q$ matrix with the associated eigenvectors.

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- **Step 5.** Let $n^* := m(q+1)$ with $m := \left\lceil \frac{n}{q+1} \right\rceil$ and denote by $\widehat{\boldsymbol{\Sigma}}^{\chi}_{n^*T}(\theta)$ the $n^* \times n^*$ spectral density matrix corresponding to X_{n^*t}.

- **Step 6.** By inverse Fourier transform of $\widehat{\mathbf{\Sigma}}^{\chi}_{n^*T}(\theta)$, estimate the autocovariance matrices $\widehat{\mathbf{\Gamma}}^{\chi}_{k}$ of the m sub-vectors

$$\boldsymbol{\chi}_t^k = (\chi_{(k-1)(q+1)+1,t} \cdots \chi_{k(q+1),t})', \quad k = 1, ..., m$$

of dimension $(q+1)$. Based on these, compute, after order identification, the Yule-Walker estimators $\hat{\mathsf{A}}^k(L)$ of the m VAR filters $\mathsf{A}^k(L)$ and stack them into a block-diagonal matrix $\hat{\mathsf{A}}_{n^*}(L)$. Compute $\hat{\mathsf{A}}_{n^*}(L)\mathsf{X}_{n^*t} = \widehat{\mathsf{Y}}_{n^*t} = \widehat{\mathsf{R}}_{n^*}\widehat{\mathsf{u}}_t + \widehat{\varepsilon}_t$

- **Step 6.** By inverse Fourier transform of $\widehat{\mathbf{\Sigma}}^{\chi}_{n*T}(\theta)$, estimate the autocovariance matrices $\widehat{\mathbf{\Gamma}}^{\chi}_{k}$ of the m sub-vectors

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- **Step 7.** Based on the first q principal volatility components of $\widehat{\mathsf{Y}}_{n*t}$, obtain estimates $\widehat{\mathsf{R}_{n*}\mathbf{u}_{t}}$ of $\mathsf{R}_{n*}\mathbf{u}_{t}$ and, via a Cholesky identification constraint the estimates $\widehat{\mathsf{R}}_{n*}$ and $\widehat{\mathbf{u}}_{t}$ of R_{n*} and $\mathbf{u}_{t}$; then, an estimate of the impulse-response function is $\widehat{\mathsf{B}}_{n*}(L) := [\hat{\mathsf{A}}_{n*}(L)]^{-1}\widehat{\mathsf{R}}_{n*}$.

- **Step 8.** Repeat Steps 2 through 7 M times: the final estimates (denoted as $\widehat{\mathsf{R}}_n$, $\widehat{\mathsf{u}}_t$, and $\widehat{\mathsf{B}}_n$) are obtained by averaging the estimates $\widehat{\mathsf{R}}_{n^*}$, $\widehat{\mathsf{u}}_t$, and $\widehat{\mathsf{B}}_{n^*}$ associated with each iteration. Let $\widehat{\chi}_{nt} := \widehat{\mathsf{B}}_n(L)\widehat{\mathsf{u}}_t$ and $\hat{\xi}_{nt} := \mathsf{X}_{nt} - \widehat{\chi}_{nt}$.

- **Step 8.** Repeat Steps 2 through 7 M times: the final estimates (denoted as $\widehat{\mathsf{R}}_n$, $\widehat{\mathsf{u}}_t$, and $\widehat{\mathsf{B}}_n$) are obtained by averaging the estimates $\widehat{\mathsf{R}}_{n^*}$, $\widehat{\mathsf{u}}_t$, and $\widehat{\mathsf{B}}_{n^*}$ associated with each iteration. Let $\widehat{\chi}_{nt} := \widehat{\mathsf{B}}_n(L)\widehat{\mathsf{u}}_t$ and $\hat{\xi}_{nt} := \mathsf{X}_{nt} - \widehat{\chi}_{nt}$.
- **Step 9.** The estimator of $\mathrm{V}(\mathsf{X}_t|\mathcal{F}_{t-1})$ is given by

$$\widehat{\mathrm{V}}(\mathsf{X}_t|\mathcal{F}_{t-1}) := \widehat{\mathsf{R}}\widehat{\mathrm{V}}(\hat{\mathsf{u}}_t|\mathcal{F}_{t-1})\widehat{\mathsf{R}}' + \widehat{\mathrm{V}}(\hat{\xi}_t|\mathcal{F}_{t-1}). \quad (9)$$

Common shocks:

- The common shocks $\mathbf{u}_t$ are assumed to be a q-dimensional stationary stable-by-aggregation MGARCH process (examples of stable-by-aggregation MGARCH are the full VECH and full BEKK).
- Estimators for BEKK or VECH are quite unstable and report several issues even in small dimensions.
- In practice, we use a DCC model (numerical experiments support our choice) instead of BEKK or VECH estimators.

Idiosyncratic components:

- Idiosyncratic components $\xi_t = (\xi_{1t}, \ldots, \xi_{nt})$ can be modelled as independent GARCH-type processes as in Hallin and Trucíos (2021) or can be modelled as a full matrix as in Trucíos et al. (2023),
- Although a full matrix does not completely escape from the curse of dimensionality, it remains implementable via the procedure of Engle et al. (2019).

Our method extends

- Alessi et al. (2009) [European Central Bank WP Series],
- Li et al. (2016) [Journal of Business & Economic Statistics]
- Barigozzi and Hallin (2020) [Journal of Econometrics],
- De Nard et al. (2021) [Journal of Econometrics]
- Trucíos et al. (2023) [Journal of Business & Economic Statistics]

Empirical Application

Let H_{T+1} be the conditional covariance matrix of vector returns at time $T+1$ and $\omega = (\omega_1, \cdots, \omega_N)$ be the portfolio weights.

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We are interesting in portfolios with minimum risk subject to some constraints, *i.e.*

$$\text{Min:} \quad \omega' H_{T+1} \omega$$

subject to $\omega_i \geq 0$ and $\sum_{i=1}^{N} \omega_i = 1$

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This portfolio is called **the minimum variance portfolio with no short sales** and is of particular interest for investors.

- **Minimum variance portfolios**
- 656 stocks entering the composition of the S&P 500, NASDAQ100 and AMEX (only series with no missing values were considered).
- Stocks traded from January 2, 2011 through June 29, 2018 (T=1884).
- **Rolling Window scheme:** A window size of 750 days is used for estimation, which represents a concentration ratio of $656/750 = 0.875$; the out-of-sample period was set to 1134 days.

At time $t = 750, \ldots, 1883$ (1134 time points), the one-step ahead conditional covariance matrix is estimated and used to obtain the optimal portfolio allocation weights.

$$\widehat{\boldsymbol{\omega}}_{t+1|t} = \operatorname*{argmin}_{\boldsymbol{\omega}} \boldsymbol{\omega}' \widehat{\Sigma}_{t+1|t} \boldsymbol{\omega},$$

subject to the restrictions $\omega_i \geq 0$ and $\sum_{i=1}^{n} \omega_i = 1$.

At time $t = 750, \ldots, 1883$ (1134 time points), the one-step ahead conditional covariance matrix is estimated and used to obtain the optimal portfolio allocation weights.

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subject to the restrictions $\omega_i \geq 0$ and $\sum_{i=1}^{n} \omega_i = 1$.

Then, the resulting (out-of-sample) portfolio return is given by

$$r_{p,t+1} := \sum_{i=1}^{n} \widehat{\omega}_{i;t+1|t} r_{i,t+1}$$

To evaluate the out-of-sample portfolio performance, we use three measures:

- SD: Annualized standard deviation (standard deviation of the out-of-sample portfolio returns $\times\sqrt{252}$);
- IR: Information ratio ($IR := AV/SD$), with AV being the annualized average portfolio (average out-of-sample portfolio returns $\times$ 252);

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- **SD**: Annualized standard deviation (standard deviation of the out-of-sample portfolio returns $\times\sqrt{252}$);
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To assess the statistical differences between our proposal and its competitors we perform the test of Ledoit and Wolf (2011) (for SD) and Ledoit and Wolf (2008) (for IR).

Empirical Application

Competitors:

- **1/n**: equal-weighted portfolio strategy advocated by DeMiguel et al. (2009).
- **POET**: the estimator proposed by Fan et al. (2013).
- **NL**: the nonlinear shrinkage estimator of Ledoit and Wolf (2012).
- **RM2006**: the RiskMetrics 2006 methodology of Zumbach (2007).
- **DCC**: the DCC model with composite likelihood of Pakel et al. (2021).
- **OGARCH**: the orthogonal GARCH model of Alexander and Chibumba (1996).
- **GPVC**: generalized principal volatility components of Li et al. (2016).

Competitors:

- PCA4TS: principal component analysis for second-order stationary vector time series of Chang et al. (2018).
- PCA-MGARCH: as used in our simulation study.
- ABC: the procedure of Alessi et al. (2009) based on the general dynamic factor model with finite-dimensional factor space.
- 2GDFM4V: the two-step GDFM procedure for volatilities proposed by Barigozzi and Hallin (2016, 2017, 2020); does not consider conditional cross-covariances.
- GDFM-CHF-DCC: our previous proposal Trucíos et al. (2023)

Table 1: Out-of-sample portfolio performance

	SD	IR
1/n	11.5067 (15)	0.5015 (15)
POET	4.6116 (10)	1.2146 (8)
NL	4.6152 (11)	1.0217 (12)
RM2006	4.5446 (8)	1.2327 (7)
DCC	5.9901 (13)	1.1502 (9)
DCC-NL	**3.9358 (2)**	**1.8371 (2)**
OGARCH	4.4551 (6)	1.1051 (11)
GPVC	4.5889 (9)	1.0022 (13)
PCA4TS	4.7256 (12)	1.1364 (10)
PCA-MGARCH	4.4111 (5)	1.4965 (5)
AFM-DCC-NL	**3.9472 (3)**	**1.9764 (1)**
ABC	4.5313 (7)	1.4404 (6)
2GDFM4V	10.2431 (14)	0.7992 (14)
GDFM-CHF-DCC	**3.9115 (1)**	**1.7065 (4)**
GDFM-PVC	**3.9931(4)**	**1.7507 (3)**

Our method outperform all previously mentioned procedures and performs statistically equal than DCC-NL, AFM-DCC-NL and GDFM-CHF-DCC.

Our method outperform all previously mentioned procedures and performs statistically equal than **DCC-NL**, **AFM-DCC-NL** and **GDFM-CHF-DCC**.

- **DCC-NL**: as proposed by Engle et al. (2019); combines the DCC and NL procedures.
- **AFM-DCC-NL**: approximate factor model as in De Nard et al. (2021) but with unobservable common components obtained by classical PCA. The covariance matrix of the idiosyncratic components is estimated using the DCC-NL procedure.
- **GDFM-CHF-DCC**: our previous proposal Trucíos et al. (2023)

Conclusions

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- Based on the one-sided estimation procedures of Forni et al. (2015, 2017), Barigozzi and Hallin (2020) and Li et al. (2016), we propose a forecasting method for the conditional covariance matrix in high-dimensional time series.
- Our proposal performs as good as the cutting-edge procedures of Engle et al. (2019), De Nard et al. (2021) and Trucíos et al. (2023)
- The initial results are quite encouraging

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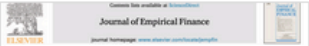

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